Geometry Notes

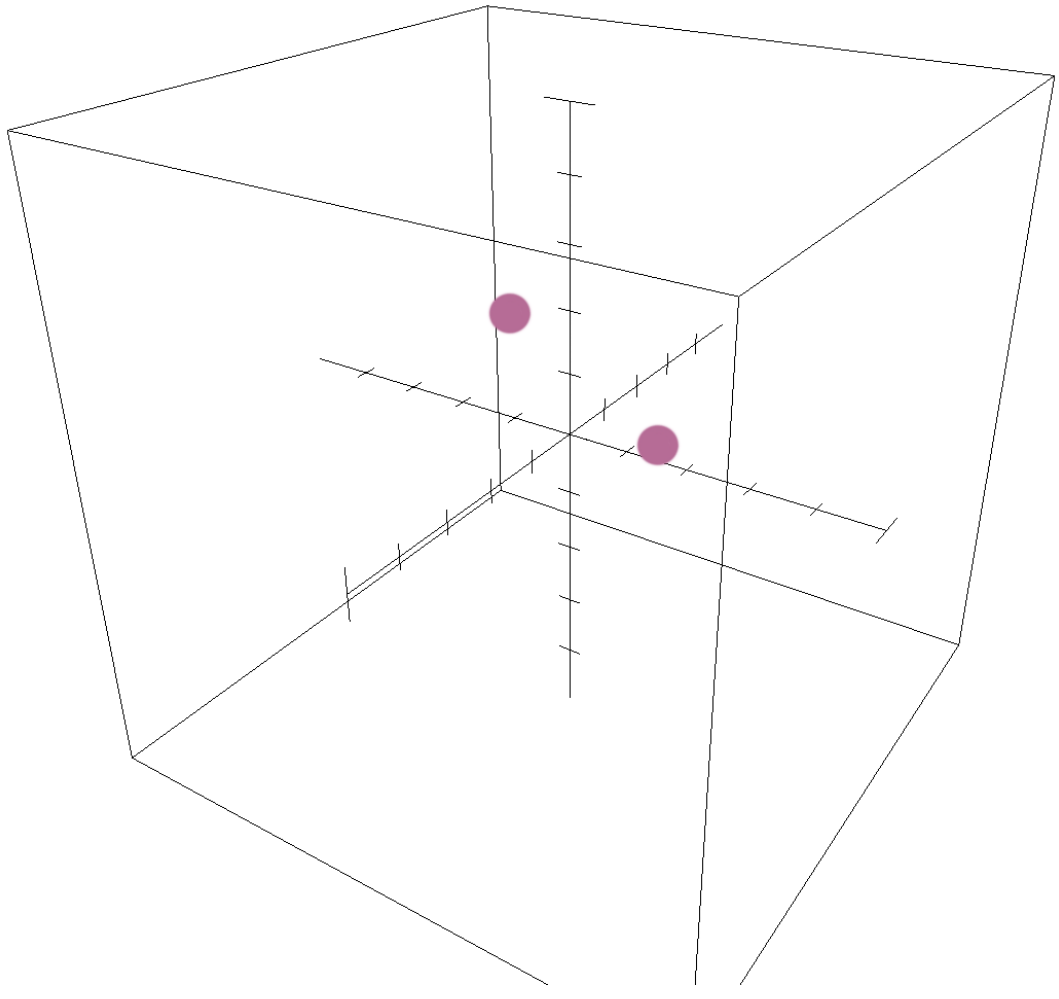
krista king
MATH

Midpoint of a line segment in three dimensions

In this lesson we'll look at how to find the midpoint of a line segment in three dimensions when we're given two three-dimensional points.

Points

A point is a representation of a location in space. Its symbol is a single letter, and it's represented as a dot. When we're talking about a three-dimensional point, we have three locations: the x-axis, the y-axis and the z-axis. So a point is represented as (x, y, z). Here are two points A and B plotted in three-dimensional space:



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Midpoint formula

We can use the midpoint formula for three dimensions to find the middle of the line segment that connects two points, which is

$$m = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

where $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$.

Let's work through an example.

Example

Find the midpoint of a line segment joining points P_1 and P_2.

$$P_1 = (4, -6, 8)$$

$$P_2 = (4, 3, -5)$$

We'll use the midpoint formula for the midpoint between points in three dimensions.

$$m = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

where $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$. We'll plug in the points we've been given, using $P_1 = (4, -6, 8)$ and $P_2 = (4, 3, -5)$.

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$$m = \left(\frac{4+4}{2}, \frac{-6+3}{2}, \frac{8+-5}{2}\right) = \left(\frac{8}{2}, \frac{-3}{2}, \frac{3}{2}\right) = (4, -1.5, 1.5)$$

Let's work through a different type of example.

Example

Find point A if m is the midpoint of $\overline{AB}$.

$$m = (4.5, -3.5, 3)$$

$$B = (2, -4, 8)$$

Let's use the midpoint formula and set up what we know.

$$m = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$(4.5, -3.5, 3) = \left(\frac{x_1 + 2}{2}, \frac{y_1 + (-4)}{2}, \frac{z_1 + 8}{2}\right)$$

Using the fact that these two points are equivalent, we can set up three equations:

$$4.5 = \frac{x_1 + 2}{2}$$

$$-3.5 = \frac{y_1 + (-4)}{2}$$

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$$3 = \frac{z_1 + 8}{2}$$

Solving each equation gives us

$$4.5 = \frac{x_1 + 2}{2}$$

$$2(4.5) = x_1 + 2$$

$$9 = x_1 + 2$$

$$7 = x_1$$

and

$$-3.5 = \frac{y_1 + (-4)}{2}$$

$$2(-3.5) = y_1 - 4$$

$$-7 = y_1 - 4$$

$$-3 = y_1$$

and

$$3 = \frac{z_1 + 8}{2}$$

$$2(3) = z_1 + 8$$

$$6 = z_1 + 8$$

$$-2 = z_1$$

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So the midpoint is $A = (7, -3, -2)$.

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